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VISCOSITY EFFECT ON REFLECTION AND TRANSMISSION COEFFICIENTS BETWEEN TWO MICROPOLAR VISCOTHERMOELASTIC HALF SPACES WITH THREE-PHASE-LAGS

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ABSTRACT
The present investigation is concerned with reflection and transmission of plane waves at an interface between two micropolar viscothermoelastic half spaces with different micropolarity and viscoelastic properties. The three-phase-lags theory of thermoelasticity developed by Roychoudhuri[12] is used to study the phenomena. The reflection and transmission coefficients of longitudinal displacement wave (LD-wave), thermal wave (T-wave) and two coupled transverse displacement and microrotational waves (CD-I wave and CD-II wave) have been derived for different incident waves. The amplitude ratios of different reflected and transmitted waves with angle of incidence are obtained for different theories of viscothermoelasticity i.e. three-phase-lags model, dual-phase-lags model and GN typeIII model. The Viscosity effect on the amplitude ratio are depicted graphically

KEYWORDS: Micropolar viscothermoelasticity, Three-phase-lagthermoelasticity, Amplitude ratios, Reflection, Transmission

I. INTRODUCTION
The classical uncoupled theory of thermoelasticity predicts two phenomenon not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms, second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves. Biot [1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories are of parabolic type predicting infinite speeds of propagation for heat waves contrary to physical observations. .
Hetnarski and Ignaczak in their survey article [2] examined five generalizations of the coupled theory and obtained a number of important analytical results.

The first is due to Lord and Shulman (L-S) [3], introduced the theory of generalized thermoelasticity which generalizes, the Fourier law of heat conduction. They have replacedFourier law by the Maxwell-Cattaneo law and introduce one relaxation time.The modified heat equation of this theory is of wave type, therefore it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory namely: the equation of motion and constitutive relations, remain the same as those for the coupled and uncoupled theories.

The second generalization is given by Green and Lindsay [4] (G-L theory), in which, the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times into consideration. This theory is also known as the theory of thermoelasticity with two relaxation times or the theory of temperature rate dependent thermoelasticity.

The third generalization of the coupled theory is known as the low-temperature thermoelasticity introduced by Hetnarski and Ignaczak [5] (H-I theory). This model is characterized by a system of non-linear field equations.
The fourth generalization is known as the thermoelasticity without energy dissipation proposed by Green and Naghdi[6].This is known as GN theory of type II, in which, the Fourier law is replaced by a heat flux rate temperature gradient relation. The heat equation in this model does not contain the temperature rate.Therefore

the solution represents an undamped thermoelastic wave. The so called Green-Naghdi theory of type III, can be derived from Green and Naghdi [7,8].

The fifth generalization to the thermoelasticity theory is known as the dual-phase-lag thermoelasticity developed by Tzou [9] and Chandrasekharaiah [10].Tzou [9] proposed a generalized heat conduction law, referred as heat conduction law with dual-phase-lags, in which microstructural effects in the heat transfer mechanism have been considered in the macroscopic formulation by taking into account that phonon-electron interactions on the macroscopic level causes a delay in the increase of the lattice temperature. A corresponding thermoelastic model with two-phase-lags was reported by Chandrasekharaiah [10]. In the models [9,10], two different phase-lags, i.e., one for the heat flux vector and other for the temperature gradient have been introduced in the Fourier's law. The phase-lag of heat flux vector is interpreted as the relaxation time due to fast transient effects of thermal inertia and the phase-lag of temperature gradient is interpreted as the delay time caused due to the microstructural interactions, a small scale effect of heat transport in space, such as phonon-electron interaction or phonon scattering. One dimensional thermoelastic wave propagation in an elastic half-space in the context of dual-phase model was studied by Roychoudhuri [11].

Roychoudhuri [12] has established a generalized mathematical model of a coupled thermoelasticity theory that includes three-phase-lags in the heat flux vector, the temperature gradient and in the thermal displacement gradient. The more general model reduces to the previous models as special cases. According to this model $\boldsymbol{q}(P,t+\tau_q) = -[K_1^*\nabla T(P,t+\tau_T) + K_1\nabla v(P,t+\tau_v)]$, where $\nabla v(\dot{v} = T)$ is the thermal displacement gradient, $\boldsymbol{q}$ is the heat flux vector, T is the thermodynamic temperature, K_1^* is the thermal conductivity and K_1 is the rate of thermal conductivity and characteristic of the theory of thermoelasticity of GN-III and the thermoelasticity with three-phase-lags.

Three-phase-lag model finds its applications in the problems of nuclear boiling, exothermic catalytic reactions, photon-electron interactions, phonon-scattering etc. In this model the delay time τ_q captures the thermal wave behavior (a small scale response in time), the phase-lag τ_T captures the effect of phonon-electron interactions (a microscopic response in space), the other delay time τ_v is effective.Inthe three-phase-lag model, the thermal displacement gradient is considered as a constitutive variable whereas in the conventional thermoelasticity theory temperature gradient is considered as a constitutive variable.

The theory of micropolar elasticity developed by Eringen [13] gives consideration to the microstructure. Micropolar theory is useful in structure materials with a fibrous, lattice or granular micropolar structure. The main difference of micropolar elastic material from the classical elastic material is that each point has extra rotational degrees of freedom independent of translation and the material can transmit couple as well as usual force stress.

The linear theory of micropolar thermoelasticity has been developed by extending the theory of micropolar continua by Erigen [14,15] and Nowacki[16] . Dost and Taborrok [17] presented the generalized thermoelasticity by using Green and Lindsay theory. A heat flux dependent micropolar thermoelsticity was developed by Chandrasekharaiah [18]. Boschi and Iesan [19] extended a generalized theory of micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speed.

The problems of reflection and transmission at the boundary surface of micropolar elastic solid half spaces have been investigated by different authors e.g. Tomar and Gogna [20, 21, 22], Hsia and Cheng [23], Hsia et al. [24], Kumar and Barak [25], Kumar et al. [26, 27], Kumar et al. [28].

Yaqin et al. [29] discussed the reflection and refraction of micropolar magneto-thermoviscoelastic waves at the interface between two micropolar viscoelastic media.Kunal and Marin [30] discussed the reflection and transmission of waves for imperfect boundary between two heat conducting micropolar thermoelastic solids.Kumar et al. (31) discussed the effect of two temperature and anisotropy in an axisymmetry problem in transversely isotropic thermoelastic solid without energy dissipation and with two temperature..Kumar et al(32) studied fundamental solution in micropolar viscothermoelastic solids with voids. Othman and Ezaira (33)

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studied Two temperature generalized thermoelastic rotating medium with voids and initial stress. Ailawalia et al. (34) studied the Plane strain problem in a rotating microstretch thermoelastic solid with microtempratutres Several researchers investigated various problems using dual-phase-lags and three-phase-lags theory of thermoelasticity e.g.Quintanilla [35, 36], Kanoria and Mallik[37], Mukhopadhyay and Kumar [38], Mukhopadhyay et al. [39], Kumar and Chawla [40], Banik and Kanoria [41].Othman et al. [42] studied the effect of rotation and intial stress on generalized micropolar thermoelastic medium with three phase lag. Marin et al. [43] studied a mixed intial-boundary value problem to modeling a three- phase-lag dipolar thermoelastic body.

The purpose of the present investigation is to study the reflection and transmission of plane waves i.e. Longitudinal displacement wave (LD-wave), thermal wave (T-wave), two coupled transverse displacement and microrotational waves(CD-I wave and CD-II wave) at an interface of micropolar viscothermoelastic half spaces for three-phase-lags model for different incident waves. Effects of viscosity are depicted graphically on the amplitude ratios of reflected and transmitted waves for a particular model.

II. BASIC EQUATIONS

Following Eringen [13] and Roychoudhuri [12], the field equations in homogeneous, isotropic, micropolar viscoelastic medium in the context of three-phase-lags model , without body forces, body couples and heat sources, are given by

$$(\lambda_1+\mu_1)\nabla(\nabla.\boldsymbol{u})+(\mu_1+K_1)\nabla^2(\boldsymbol{u})+K_1(\nabla\times\boldsymbol{\varphi})-\nu\nabla T=\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{1}$$

$$(\alpha_1+\beta_1)\nabla(\nabla.\boldsymbol{\varphi})+\gamma_1(\nabla^2\boldsymbol{\varphi})+K_1(\nabla\times\boldsymbol{u})-2K_1\boldsymbol{\varphi}=\rho j\frac{\partial^2\boldsymbol{\varphi}}{\partial t^2}, \tag{2}$$

$$[K_0(1+\tau_v\frac{\partial}{\partial t})+K_1^*\frac{\partial}{\partial t}(1+\tau_T\frac{\partial}{\partial t})]\nabla^2T=(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})[\rho c^*\frac{\partial^2}{\partial t^2}T+\nu T_0\frac{\partial^2}{\partial t^2}(\nabla.\boldsymbol{u})], \tag{3}$$

and the constitutive relations are

$$t_{ij}=\lambda_1u_{r,r}\delta_{ij}+\mu_1(u_{i,j}+u_{j,i})+K_1(u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu T\delta_{ij}, \tag{4}$$

$$m_{ij}=\alpha_1\phi_{r,r}\delta_{ij}+\beta_1\phi_{i,j}+\gamma_1\phi_{j,i}\,,\qquad i,j,r=1,2,3$$

Where

$$\lambda_I=\lambda+\lambda_v\frac{\partial}{\partial t},\mu_I=\mu+\mu_v\frac{\partial}{\partial t},K_I=K+K_v\frac{\partial}{\partial t},\alpha_I=\alpha+\alpha_v\frac{\partial}{\partial t},\beta_I=\beta+\beta_v\frac{\partial}{\partial t},\gamma_I=\gamma_.+\gamma_v\frac{\partial}{\partial t}. \tag{5}$$

,

In the above equations, λ and μ are Lame's constants. K, α, β and γ are micropolar constants. t_{ij} and m_{ij} are the components of stress tensor and couple stress tensor respectively. $\boldsymbol{u}$ and $\boldsymbol{\varphi}$ are the displacement and microrotation vectors, ρ is the density , j is the microinertia, c^* is the specific heat at constant strain, K_1^* is the thermal conductivity, K_0 is the rate of thermal conductivity and characteristic of the theory of thermoelasticity of GN-III and the thermoelasticity with three-phase-lags, T is the thermodynamic temperature, T_0 is the reference temperature, $\nu=(3\lambda_1+2\mu_1+K_1)\alpha_T$, where α_T is the coefficient of linear thermal expansion, $,\lambda_I,\mu_I,K_I,\alpha_I,\beta_I,\gamma_I,$ are material constants $,\lambda_V,\mu_V,K_V,\alpha_V,\beta_V,\gamma_V$ are the material constants τ_v, τ_T and τ_q are the phase-lags of the temperature gradient, the heat flux and of thermal displacement gradient respectively, such that $\tau_v<\tau_T<\tau_q$. δ_{ij} is the Kronecker delta, ε_{ijr} is the alternating tensor and ∇^2 is the Laplacian operator.

III. FORMULATION OF THE PROBLEM

We consider two homogeneous, isotropic, micropolar, viscothermoelastic solid half spaces with three phase lags (medium M_1) and (medium M_2) in contact with each other at a plane surface, which we take as the plane $x_3=0$ of a rectangular co-ordinate system $Ox_1x_2x_3$ as shown in fig. 1. Let us consider plane waves in the x_1x_3-plane with wave front parallel to x_2-axis.We write all the variables without bar in medium M_1 and attach bar to denote the variables in medium M_2 as shown in Fig.1.

The displacement and microrotation components for two dimensional problem are taken as

$$\boldsymbol{u}=(u_1(x_1,x_3),0,u_3(x_1,x_3)), \qquad \boldsymbol{\varphi}=(0,\phi_2(x_1,x_3),0) \tag{6}$$

We define the dimensionless quantities as

$$x_1'=\frac{x_1}{L},\quad x_3'=\frac{x_3}{L},\quad (u_1',u_3',\overline{u_1'},\overline{u_3'})=(u_1,u_3,\overline{u_1},\overline{u_3})\frac{1}{L},\quad (\phi_2',\overline{\phi_2'})=(\phi_2,\overline{\phi_2})\frac{\lambda}{\nu T_0},$$

$$(t',\overline{t'})=(t,\overline{t})\frac{c_1}{L},\quad (T',\overline{T'})=(T,\overline{T})\frac{1}{T_0},\quad (t_{ij}',\overline{t_{ij}'})=(t_{ij},\overline{t_{ij}})\frac{1}{\nu T_0},\quad (m_{ij}',\overline{m_{ij}'})=(m_{ij},\overline{m_{ij}})\frac{1}{L\nu T_0},$$

$$(\tau_q',\overline{\tau_q'})=(\tau_q,\overline{\tau_q})\frac{c_1}{L},\quad (\tau_T',\overline{\tau_T'})=(\tau_T,\overline{\tau_T})\frac{c_1}{L},\quad (\tau_v',\overline{\tau_v'})=(\tau_v,\overline{\tau_v})\frac{c_1}{L} \tag{7}$$

where

$c_1^2=\dfrac{\lambda+2\mu+K}{\rho}$, L is a parameter having dimensions of length.

By Helmholtz representation of displacement vector,we can write

$$\vec{u}=gad\phi+curl\,\vec{\psi}\quad where\ \ \phi \text{ is scalar potential } \& \ \vec{\psi} \text{ is vector potential } -------------(8)$$

Substituting the values of $\vec{u}$ from Eqs.(8) in Eqs. (1)-(3) and with the aid of (6) &(7) after simplify the equations and suppressing the primes, we obtain

$$a_3\nabla^2\phi-a_2T-a_3\frac{\partial^2\phi}{\partial t^2}=0, \tag{9}$$

$$\nabla^2\psi+a_1\phi_2-a_3\frac{\partial^2\psi}{\partial t^2}=0, \tag{10}$$

$$\nabla^2\phi_2-a_4\nabla^2\psi-a_5\phi_2-a_6\frac{\partial^2\phi_2}{\partial t^2}=0, \tag{11}$$

$$[a_7(1+\tau_v\frac{\partial}{\partial t})+\frac{\partial}{\partial t}(1+\tau_T\frac{\partial}{\partial t})]\nabla^2T=(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})[a_8\frac{\partial^2}{\partial t^2}T+a_9\frac{\partial^2}{\partial t^2}\nabla^2\phi], \tag{12}$$

where

$$a_1=\frac{K_1\nu T_0}{(\mu_1+K_1)\lambda_1},\quad a_2=\frac{\nu T_0}{\mu_1+K_1},\quad a_3=\frac{\rho c_1^2}{\mu_1+K_1},\quad a_4=\frac{K_1L^2\lambda_1}{\gamma_1\nu\ T_0},\quad a_5=\frac{2K_1L^2\lambda_1}{\gamma_1},\quad a_6=\frac{\rho jc_1^2}{\gamma_1},$$

$$a_7=\frac{K_0L}{K_1^*c_1},\quad a_8=\frac{\rho c^*c_1L}{K_1^*},\quad a_9=\frac{\nu c_1L}{K_1^*},\quad \nabla^2=\frac{\partial}{\partial x_1^2}+\frac{\partial}{\partial x_3^2},\quad K_0=\frac{c^*(\lambda+2\mu)}{4}$$

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$$\lambda_1 = \lambda(1+Q_1\frac{\partial}{\partial t}) \quad \mu_1 = \mu(1+Q_2\frac{\partial}{\partial t}), K_1 = K(1+Q_3\frac{\partial}{\partial t}), \alpha_1 = \alpha(1+Q_4\frac{\partial}{\partial t}), \beta_1 = \beta(1+Q_5\frac{\partial}{\partial t}),$$

$$\gamma_1 = \gamma(1+Q_6\frac{\partial}{\partial t}),$$

$$(Q_1, Q_2, Q_3, Q_4, Q_5, Q_6) = \frac{c_1}{L}(\frac{\lambda_v}{\lambda}, \frac{\mu_v}{\mu}, \frac{K_v}{K}, \frac{\alpha_v}{\alpha}, \frac{\beta_v}{\beta}, \frac{\gamma_v}{\gamma})$$

IV. BOUNDARY CONDITIONS

Meccanical conditions;
Continuity of stress,displacement and micropolar rotation vector components

Thermal conditions;
Comtinuity of temperature change and normal components of heat flux .

Mathematically these conditions at the boundary interface $x_3 = 0$ can be written as

$$t_{33} = \bar{t}_{33}, \quad t_{31} = \bar{t}_{31}, \quad m_{32} = \bar{m}_{32}, \quad u_1 = \bar{u}_1, \quad u_3 = \bar{u}_3, \quad \phi_2 = \bar{\phi}_2, \quad T = \bar{T},$$

$$M_1\frac{\partial T}{\partial x_3} = \overline{M_1}\frac{\partial \bar{T}}{\partial x_3} \qquad (13)$$

where

$$M_1 = [K_1(1+\tau_v\frac{\partial}{\partial t}) + K_1^*(1+\tau_T\frac{\partial}{\partial t})\frac{\partial}{\partial t}], \quad \overline{M_1} = [\overline{K_1}(1+\overline{\tau_v}\frac{\partial}{\partial t}) + \overline{K_1^*}(1+\overline{\tau_T})\frac{\partial}{\partial t}]$$

V. REFLECTION AND TRANSMISSION

We consider Longitudinal displacement wave (LD-wave), Thermal wave (T-wave), two coupled transverse displacement and microrotational waves(CD-I wave and CD-II wave) propagating through the medium M_1 which we designate as the region $x_3 > 0$ and incident at the plane $x_3 = 0$ with its direction of propagation with angle θ_0 normal to the surface. Corresponding to each incident wave, we get reflected LD-wave, T-wave, CD-I and CD-II waves in medium M_1 and transmitted LD-wave, T-wave, CD-I and CD-II waves in medium M_2.

To solve the equations (9)-(12), we take the solutions of the form

$$\{\phi, T, \psi, \phi_2\} = \{\tilde{\phi}, \tilde{T}, \tilde{\psi}, \tilde{\phi}_2\}\, e^{\iota\{k(x_1\sin\theta - x_3\cos\theta) - \omega t\}} \qquad (14)$$

such that k is the wave number, ω is the angular frequency and $\tilde{\phi}, \tilde{T}, \tilde{\psi}, \tilde{\phi}_2$ are arbitrary constants.

Substituting the values of ϕ, T, ψ, ϕ_2 from (14) in equations (9)-(12), After some algebraic calculations we obtain

$$V^4 + D_1V^2 + E_1 = 0, \qquad (15)$$

$$V^4 + D_2V^2 + E_2 = 0, \qquad (16)$$

where

$$D_1 = \frac{(a_2 a_9 - a_3 a_8)(1 - \iota\omega\tau_q - \frac{\omega^2}{2}\tau_q^2) - a_3[(1-\iota\omega\tau_v)a_7 - \iota\omega(1-\iota\omega\tau_T)]}{a_3 a_8 (1 - \iota\omega\tau_q - \frac{\omega^2}{2}\tau_q^2)},$$

$$E_1 = \frac{a_3[(1-\iota\omega\tau_v)a_7 - \iota\omega(1-\iota\omega\tau_T)]}{a_3 a_8 (1 - \iota\omega\tau_q - \frac{\omega^2}{2}\tau_q^2)},$$

$$D_2 = -(\frac{a_1 a_4}{\omega^2} + a_3)\frac{1}{a_3(a_6 - \frac{a_5}{\omega^2})} - \frac{1}{a_3}, \qquad E_2 = \frac{1}{(a_6 - \frac{a_5}{\omega^2})a_3}$$

and $V^2 = \frac{\omega^2}{k^2}$

Equations (15) and (16) are quadratic in V^2, therefore the roots of these equations give four values of V^2. Corresponding to each value of V^2 in equation (15), there exist two types of waves in medium M_1 in decreasing order of their velocities, namely LD-wave and T-wave. Similarly corresponding to each value of V^2 in equation (16), there exist two types of waves in medium M_1, namely CD-I wave and CD-II wave. Let V_1, V_2 be the velocities of reflected LD-wave, T-wave and V_3, V_4 be the velocities of reflected CD-I wave, CD-II wave in medium M_1.

In view of equation (14), the appropriate solutions of equations (9)-(12) for medium M_1 and medium M_2 are taken as

Medium M_1:

$$\{\phi, T\} = \sum_{i=1}^{2}\{1, f_i\}[S_{0i} e^{\iota\{k_i(x_1 \sin\theta_{0i} - x_3\cos\theta_{0i}) - \omega_i t\}} + P_i], \qquad (17)$$

$$\{\psi, \phi_2\} = \sum_{j=3}^{4}\{1, f_j\}[T_{0j} e^{\iota\{k_j(x_1 \sin\theta_{0j} - x_3\cos\theta_{0j}) - \omega_j t\}} + P_j], \qquad (18)$$

where

$$f_i = \frac{a_3 a_9 \omega_i^2 (1 - \iota\omega_i\tau_q - \frac{\omega_i^2}{2}\tau_q^2)}{(a_2 a_9 + a_3 a_8)(1 - \iota\omega_i\tau_q - \frac{\omega_i^2}{2}\tau_q^2) - \frac{1}{V_i^2} a_3[(1-\iota\omega\tau_v)a_7 - \iota\omega(1-\iota\omega\tau_T)]},$$

$$f_j = \frac{a_3 a_4}{\frac{1}{V_j^2} + ((a_5 - a_1 a_4)/\omega_j^2) - a_6}$$

and $P_i = S_i e^{\iota\{k_i(x_1\sin\theta_i + x_3\cos\theta_i) - \omega_i t\}}$, $\qquad P_j = T_j e^{\iota\{k_j(x_1\sin\theta_j + x_3\cos\theta_j) - \omega_j t\}}$

such that S_{0i}, T_{0j} are the amplitudes of incident (LD-wave, T-wave) and (CD-I, CD-II) waves respectively. S_i and T_j are the amplitudes of reflected (LD-wave, T-wave) and (CD-I, CD-II) waves.

Medium M_2:

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$$\{\bar{\phi}, \bar{T}\} = \sum_{i=1}^{2}\left\{1, \bar{f}_i\right\}[\bar{S}_i\, e^{\iota\left\{\bar{k}_i\left(x_1 \sin\bar{\theta}_i - x_3\cos\bar{\theta}_i\right) - \bar{\omega}_i t\right\}}], \tag{19}$$

$$\{\bar{\psi}, \bar{\phi}_2\} = \sum_{j=3}^{4}\left\{1, \bar{f}_j\right\}[\bar{T}_j\, e^{\iota\left\{\bar{k}_j\left(x_1 \sin\bar{\theta}_j - x_3\cos\bar{\theta}_j\right) - \bar{\omega}_j t\right\}}], \tag{20}$$

where $\bar{S}_i$, $\bar{T}_j$ are the amplitudes of transmitted (LD-wave, T-wave) and (CD-I, CD-II) waves respectively.
We use the following extension of the Snell's law to satisfy the boundary conditions

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3} = \frac{\sin\theta_4}{V_4} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2} = \frac{\sin\bar{\theta}_3}{\bar{V}_3} = \frac{\sin\bar{\theta}_4}{\bar{V}_4} \tag{21}$$

where $V_j = \dfrac{\omega}{k_j}$, $\bar{V}_j = \dfrac{\omega}{\bar{k}_j}$ (j =1, 2, 3, 4) at x_3 =0 (22)

Making use the values of ϕ, ψ, T and ϕ_2 from equations (17)-(20) the boundary conditions (13) and with the aid of equations (4)-(8), (21) and (22), we obtain a system of eight non-homogeneous equations as

$$\sum_{j=1}^{8} a_{ij} Z_j = Y_i \; ; \; (i = 1, 2, 3, 4, 5, 6, 7, 8) \tag{23}$$

where the values of a_{ij} are given as

$$a_{1i} = (d_1 + d_2 B_i)\frac{\omega^2}{V_i^2} + f_i \,,\; a_{1j} = d_2\frac{\omega^2}{V_j V_0}\sin\theta_0\sqrt{B_j} \,,\; a_{1k} = -\left[\left(\bar{d}_1 + \bar{d}_2 R_i\right)\frac{\omega^2}{\bar{V}_i^2} + \bar{p}_0\bar{f}_i\right],$$

$$a_{1l} = -\bar{d}_2\frac{\omega^2}{\bar{V}_j V_0}\sin\theta_0\sqrt{R_j} \,,\quad a_{2i} = -(2d_3 + d_4)\frac{\omega^2}{V_1 V_0}\sin\theta_0\sqrt{B_i}\,,$$

$$a_{2j} = (d_3 + d_4)\frac{\omega^2}{V_j^2}B_j - d_3\frac{\omega^2}{V_0^2}\sin^2\theta_0 - d_5 f_j,\quad a_{2k} = \left(2\bar{d}_3 + \bar{d}_4\right)\frac{\omega^2}{\bar{V}_i V_0}\sin\theta_0\sqrt{R_i}\,,$$

$$a_{2l} = -\left[\bar{d}_3\frac{\omega^2}{\bar{V}_j^2}\left(1 - 2\frac{\bar{V}_j^2}{V_0^2}\sin^2\theta_0\right) + \bar{d}_4\frac{\omega^2}{\bar{V}_j^2}R_j - \bar{d}_5\bar{f}_j\right],$$

$$a_{3i} = 0,\quad a_{3j} = \iota\frac{\omega}{V_j}\sqrt{B_j}\, f_j \,,\quad a_{3k} = 0,\quad a_{3l} = \iota p_1\frac{\omega}{\bar{V}_j}\sqrt{R_j}\;\bar{f}_j\,,$$

$$a_{4i} = \iota\frac{\omega}{V_0}\sin\theta_0 \,,\qquad a_{4j} = -\iota\frac{\omega}{V_j}\sqrt{B_j}\,,\qquad a_{4k} = -\iota\frac{\omega}{V_0}\sin\theta_0,\; a_{4l} = -\iota\frac{\omega}{\bar{V}_j}\sqrt{R_j}\,,$$

$$a_{5i} = \iota\frac{\omega}{V_i}\sqrt{B_i}\,,\quad a_{5j} = \iota\frac{\omega}{V_0}\sin\theta_0,\quad a_{5k} = \iota\frac{\omega}{\bar{V}_i}\sqrt{R_i},\; a_{5l} = -\iota\frac{\omega}{V_0}\sin\theta_0\,,$$

$$a_{6i} = 0,\quad a_{6j} = f_j\,,\quad a_{6k} = 0\,,\quad a_{6l} = -\bar{f}_j,\quad a_{7i} = f_i\,,\; a_{7j} = 0,\; a_{7k} = -\bar{f}_i\,,\; a_{7l} = 0\,,$$

$$a_{8i} = \iota\frac{\omega}{V_i}f_i\sqrt{B_i}\;[(1 - \iota\omega\tau_T)(-\iota\omega) + a_7(1 - \iota\omega\tau_v)],\; a_{8j} = 0,$$

$$a_{8k} = \iota\frac{\omega}{\bar{V}_i}\bar{f}_i\sqrt{R_i}\;[(1 - \iota\omega\bar{\tau}_T)(-\iota\omega) + \bar{a}_7(1 - \iota\omega\bar{\tau}_v)],\; a_{8l} = 0,$$

$$d_1=\frac{\lambda_1}{\nu T_0},\ d_2=\frac{(2\mu_1+K_1)}{\nu T_0},\ d_3=\frac{\mu_1}{\nu T_0},\ d_4=\frac{K_1}{\nu T_0},\ d_5=\frac{K_1}{\lambda_1},$$

$$\overline{d_1}=\frac{\overline{\lambda}}{\nu T_0},\ \overline{d_2}=\frac{(2\overline{\mu}+\overline{K})}{\nu T_0},\ \overline{d_3}=\frac{\overline{\mu}}{\nu T_0},\ \overline{d_4}=\frac{\overline{K}}{\nu T_0},\ \overline{d_5}=\frac{\overline{K}}{\lambda_1},\ p_1=\frac{\overline{\gamma}}{\gamma_1},\ p_2=\frac{\overline{K_1^*}}{K_1^*},\ \overline{p_0}=\frac{\overline{\nu}}{\nu},$$

$$B_i=(1-\frac{V_i^2}{V_0^2}\sin^2\theta_0),\ B_j=(1-\frac{V_j^2}{V_0^2}\sin^2\theta_0),\ R_i=(1-\frac{\overline{V_i^2}}{V_0^2}\sin^2\theta_0),\ R_j=(1-\frac{\overline{V_j^2}}{V_0^2}\sin^2\theta_0) \quad (24)$$

In the above equation (24) i =1, 2, j =3, 4, k =5, 6, and l =7, 8and

$$Z_1=\frac{S_1}{A^*},\ Z_2=\frac{S_2}{A^*},\ Z_3=\frac{T_3}{A^*},\ Z_4=\frac{T_4}{A^*},\ Z_5=\frac{\overline{S_1}}{A^*},\ Z_6=\frac{\overline{S_2}}{A^*},\ Z_7=\frac{\overline{T_3}}{A^*},\ Z_8=\frac{\overline{T_4}}{A^*} \quad (25)$$

such that Z_1, Z_2, Z_3, Z_4 are the amplitude ratios of reflected LD-wave, T-wave and coupled CD-I, CD-II waves in medium M_1 and Z_5, Z_6, Z_7, Z_8 are the amplitude ratios of transmitted LD-wave, T-wave and coupled CD-I, CD-II waves in medium M_2 and Y_i is defined in the following paragraph.

(1) For incident LD-wave:

$A^*=S_{01}$, $S_{02}=T_{03}=T_{04}=0$, $Y_1=-a_{11}$, $Y_2=a_{21}$, $Y_3=a_{31}=0$, $Y_4=-a_{41}$,
$Y_5=a_{51}$, $Y_6=a_{61}=0$, $Y_7=-a_{71}$, $Y_8=a_{81}$

(2) For incident T-wave:

$A^*=S_{02}$, $S_{01}=T_{03}=T_{04}=0$, $Y_1=-a_{12}$, $Y_2=a_{22}$, $Y_3=a_{32}=0$, $Y_4=-a_{42}$,
$Y_5=a_{52}$, $Y_6=a_{62}=0$, $Y_7=-a_{72}$, $Y_8=a_{82}$

(3) For incident CD-I wave:

$A^*=T_{03}$, $S_{01}=S_{02}=T_{04}=0$, $Y_1=a_{13}$, $Y_2=-a_{23}$, $Y_3=a_{33}$, $Y_4=a_{43}$,
$Y_5=-a_{53}$, $Y_6=-a_{63}$, $Y_7=a_{73}=0$, $Y_8=a_{83}=0$

(4) For incident CD-II wave:

$A^*=T_{04}$, $S_{01}=S_{02}=T_{03}=0$, $Y_1=a_{14}$, $Y_2=-a_{24}$, $Y_3=a_{34}$, $Y_4=a_{44}$,
$Y_5=-a_{54}$, $Y_6=-a_{64}$, $Y_7=a_{74}=0$, $Y_8=a_{84}=0$

VI. PARTICULAR CASES

Case I Dual-phase-lags micropolar viscothermoelasticity

If we take $\tau_\nu=0$, $\overline{\tau_\nu}=0$, $K_1=0$, $\overline{K_1}=0$ in eq. (23), then we obtain the amplitude ratios at the boundary of two micropolar viscothermoelastic solid half spaces in the context of dual-phase-lags thermoelasticity. The values of a_{ij} given by equation (24) with the following changed values

$$a_{8i}=\frac{\omega^2}{V_i}f_i\sqrt{B_i}\ (1-\iota\omega\tau_T),\ a_{8k}=\frac{\omega^2}{\overline{V_i}}\overline{f_i}\sqrt{R_i}\ (1-\iota\omega\overline{\tau_T})$$

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Case II: **GN type III micropolar viscothermoelasticity**

If we take $\tau_v = \tau_T = \tau_q = 0, \overline{\tau_v} = \overline{\tau_T} = \overline{\tau_q} = 0$ in eq. (23),then we obtain the amplitude ratios at the boundary of two micropolar viscothermoelastic solid half spaces using GN type III thermoelasticity and the values of a_{ij} are given by equation (24) with the changed values of a_{ij} as

$$a_{8i} = \iota \frac{\omega}{V_i} f_i \sqrt{B_i}\ (a_7 - \iota\omega),\ a_{8k} = \iota \frac{\omega}{\overline{V_i}} \overline{f_i} \sqrt{R_i}\ (\overline{a_7} - \iota\omega)$$

$$Q_i\,(i = 1,2,3,4,5,6) \to 0$$

Case III: If in equation(23) , we obtain the expression of the amplitude ratios at the plane boundary of two micropolarthermoelastic solid half spaces with three phase lags.

VII. NUMERICAL RESULTS AND DISCUSSION

Numerical computations has been done with the following values of relevant parameters for both the half spaces.

The values of micropolar constants for medium M_1 are taken from Eringen [44]:

$\lambda = 9.4 \times 10^{10}\, Nm^{-2}$, $\mu = 4.0 \times 10^{10}\, Nm^{-2}$, $K = 1.0 \times 10^{10}\, Nm^{-2}$, $\gamma = 7.79 \times 10^{-10}\, N$,
$j = 0.002 \times 10^{-17}\, m^2$, $\rho = 1.74 \times 10^3\, Kgm^{-3}$

and thermal parameters are taken from Dhaliwal and Singh [45]:

$\nu = 2.68 \times 10^4\, Nm^{-2}K^{-1}$, $c^* = 1.04 \times 10^3\, NmKg^{-1}K^{-1}$, $\tau_v = 0.05,\ \tau_T = 0.1,\ \tau_q = 0.5,$
$T_0 = 0.298K$, $K_1^* = 1.7 \times 10^2\, Nsec^{-1}K^{-1}$, $\omega = 1$

Following Gauthier [46], the values of micropolar constants for medium M_2 are taken as:

$\overline{\lambda} = 7.59 \times 10^{10}\, Nm^{-2}$, $\overline{\mu} = 0.0189 \times 10^{12}\, Nm^{-2}$, $\overline{j} = 0.00196 \times 10^{-17}\, m^2$,
$\overline{K} = 0.0149 \times 10^9\, Nm^{-2}$, $\overline{\gamma} = 2.68 \times 10^{-5}\, N$, $\overline{\rho} = 2.19 \times 10^3\, Kgm^{-3}$,

Thermal parameters for the medium M_2 are taken as:

$\overline{T_0} = 0.0296K$, $\overline{K_1^*} = 1.2 \times 10^2\, Nsec^{-1}K^{-1}$, $\overline{\tau_v} = 0.015,\ \overline{\tau_T} = 0.02,\ \overline{\tau_q} = 0.05,$
$\overline{\nu} = 0.02603 \times 10^8\, Nm^{-2}K^{-1}$, $\overline{c^*} = 9.21 \times 10^2\, JKg^{-1}K^{-1}$, $Q_1 = 5, Q_2 = 10, Q_3 = 15, Q_4 = 20.$
$\overline{Q_1} = 3, \overline{Q_2} = 5, \overline{Q_3} = 8, \overline{Q_4} = 10$

The values of amplitude ratios have been computed at different angles of incidence.

The variations of amplitude ratios for micropolar thermoelastic (viscoelastic) solid half spaceswith three-phase-lags model (VSTP), micropolar thermoelastic solid half spaces with three-phase-lags model (TP) have been shown by solid line and small dashes line respectively in figs. 2-4.

Incident LD-Wave

Variations of amplitude ratios $|Z_i|; 1 \le i \le 8$ with the angle of incidence θ_0, for incident LD-wave are shown in Figs. 2(a) through 2(h).

Fig.2(a) shows that the values of amplitude ratio $|z_1|$ for VSTP decreases monotonically for $0^0 \le \theta_0 \le 50^0$ and after that it increases with the increase of the value of θ_0 but the values of amplitude ratio for TP are

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oscillatory with the increase in θ_0. Due to viscosity effect the values for TP remain more than the value forVSTP in the whole range.

Fig.2(b) depicts that the values of $|z_2|$ for VSTP and TP are oscillatory for the whole range.The values of amplitude ratio for VSTP remain less in comparison than the values of amplitude ratio for TP in the range $0^0 \le \theta_0 \le 85^0$ and after that the values of amplitude ratio for TP and for VSTP approaches to coincide.

Fig.2(c) shows that the values of amplitude ratio $|z_3|$ both for TP and VSTP increase monotonically for $0^0 \le \theta_0 \le 55^0$ and after that the value of amplitude ratio decreases as θ_0 increases.The value of amplitude ratio for TP is more than the value of amplitude ratio for VSTP in the range $0^0 \le \theta_0 \le 75^0$ and after that the behavior is reversed which reveals the effect of viscosity.

It is noticed from fig.2(d) that the behaviour of variation of $|z_4|$ is similar to that of amplitude ratio $|z_3|$ with difference in magnitude values.

It is depicted from fig.2(e) the values of $|z_5|$ for TP and VSTP decrease in the range $0^0 \le \theta_0 \le 58^0$ and when $\theta_0 \ge 58^0$ the values of amplitude ratio increase in the small range and then decrease.The values of amplitude ratio for VSTP remains more in comparison than the values for TP in the whole range.

Fig .2(f)shows that the values of amplitude ratio $|z_6|$ for TP decrease with slight oscillation.For VSTP

The values of amplitude ratio decrease strictly in the range $0^0 \le \theta_0 \le 55^0$.and values of amplitude ratio increase for $55^0 \le \theta_0 \le 75^0$ and then values decrease in the rest of range.The values of amplitude ratio for TP is more than the values for VSTP in the range $15^0 \le \theta_0 \le 85^0$.when $\theta_0 \ge 85^0$ the values of amplitude ratio for VSTP and for TP approaches coincide.

Fig.2(g) shows that the values of amplitude ratio $|z_6|$ for VSTP increase with slight oscillation in the range $0^0 \le \theta_0 \le 70^0$ and maximum value is attained between $70^0 \le \theta_0 \le 80^0$ after that the values of amplitude ratio decrease with the increase in angle of incidence. the values of amplitude ratio $|z_6|$ for TP increase in the range $0^0 \le \theta_0 \le 40^0$ and maximum value is attained between $40^0 \le \theta_0 \le 60^0$ after that the values of amplitude ratio decrease.the values of amplitude ratio for TP is more than the values of amplitude for VSTP in the range $0^0 \le \theta_0 \le 75^0$ after that behavior is reversed.

It is noticed from fig. 2(h)that behavior of variation of $|Z_8|$ is similar to that of $|Z_7|$ with difference in their magnitude values.The values for TP and forVSTP are magnified by multiplying by 10^4.

Incident T-Wave

Variation of amplitude ratios $|Z_i|;1 \le i \le 8$, with the angle of incidence θ_0, for incident T-wave are shown in Figs. 3(a) through 3(h).

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Fig. 3(a) shows that the values of $|Z_1|$ for VSTP oscillate in the range $0^0 < \theta_0 < 52^0$ and after that value of amplitude ratio decrease further for $\theta_0 \geq 52^0$ The values of $|Z_1|$ for TP oscillates in the range $0^0 < \theta_0 < 70^0$ and then decrease with further increase in θ_0.Due to viscosity the values of amplitude for VSTP is more than the values for TP in the range $10^0 < \theta_0 < 75^0$. *when* $\theta_0 \geq 75^0$. the values for TP and for VSTP approaching coincide.

Fig.3(b) shows that the values of amplitude ratio $|Z_2|$ for TP and VSTP decrease in the initial range and then increase with increase in θ_0. The values of amplitude ratio for TP remain less in comparison than the values for VSTP when $\theta_0 \geq 10^0$ which shows the effect of viscosity.

It is evident from fig. 3(c) that the values of $|Z_3|$ for TP increase in the interval $0^0 < \theta_0 < 51^0$, while the values for VSTP increase in the range $0^0 < \theta_0 < 55^0$ and then decrease in the further range. The values for TP remain more than the values for VSTP in the whole range and maximum value of amplitude ratio are attained in the range $45^0 < \theta_0 < 55^0$.

Fig. 3(d) depicts that behavior of variation of $|Z_4|$ for TP and VSTP is similar in the whole range with difference in magnitudes. It is noticed that the values for TP remain more than the values for VSTP in the whole range.

Fig. 3(e) shows the variation of amplitude ratio $|Z_5|$ with angle of incidence. It shows that the values for VSTP oscillate in the range $0^0 < \theta_0 < 55^0$ and value of amplitude ratio decrease with further increase in θ_0. The values for TP oscillates in the range $0^0 < \theta_0 < 65^0$ and value decreases with further increase in θ_0.

Fig. 3(f) presents the variation of $|Z_6|$ with angle of incidence θ_0. It is noticed that the value of amplitude ratio for VSTP decreases as θ_0 increases for the whole range. while the value for TP decreases when $0^0 < \theta_0 < 56^0$ after that value of amplitude ratio oscillates slightly for further Increase in θ_0. The values for VSTP remain more than the values for TP in the whole range.

It is noticed from the fig.3(g) behavior of variation of $|Z_7|$ for TP and VSTP is similar with difference in magnitude. The values of amplitude ratiofor TP are more in comparison than the value for VSTP in the whole range . These variations are shown in fig. 3(g).The values for TP and VSTP are magnified by multiplying by 10.

Fig. 3(h) shows that the behavior of variation of $|Z_8|$ is similar as that of $|Z_7|$ with difference in their magnitude values. The values of amplitude ratio for TP remain more than the values for VSTP in the whole range which reveals the effect of viscosity.

Incident CD-I Wave

Variations of amplitude ratios $|Z_i|; 1 \leq i \leq 8$, with the angle of incidence θ_0 , for incident CD-I wave are shown in figs. 4(a) through 4(h).

Fig. 4(a) shows that the value of $|Z_1|$ for VSTP increases in the range $0^0 < \theta_0 < 85^0$ after that the value of amplitude ratio decreases with further increase in θ_0. the values of $|Z_1|$ for TP increases with slight oscillation in the whole range.the values of amplitude ratio for VSTP is more than the values for TP in the whole range.which shows the effect of viscosity.

Fig.4(b) shows that the values of amplitude ratio $|Z_2|$ for TP and VSTP increase with the increase in the θ_0. The values for VSTP remain less than the values for TP for the range $0^0 < \theta_0 < 40^0$ and in the remaining range, the behavior is reversed, that depicts the effect of three-phase-lags.

It is seen from fig. 4(c) that the values of $|Z_3|$ for TP increase in the whole range, while the values of amplitude ratio for VSTP increase in the range $0^0 < \theta_0 < 55^0$ and then decrease in the further range. The values of $|Z_3|$ for TP remain less than the values for VSTP in the whole range and maximum value of $|Z_3|$ for VSTP is attained in the range. $45^0 < \theta_0 < 60^0$.

Fig. 4(d) depicts that behavior of variation of $|Z_4|$ for TP and VSTP . It is noticed that the values of amplitude ratio for TP remain more than the values for VSTP in the whole range. The values of amplitude ratio for VSTP intially remains constant as θ_0 increases after that values of amplitude ratio increase slowly as angle of incidence increases.The value of amplitude ratio for TP increases monotonically as angle of incidence increases. Fig. 4(e) shows the variation of amplitude ratio $|Z_5|$ with angle of incidence. It shows that the values of $|Z_5|$ for VSTP increase in the range $0^0 < \theta_0 < 55^0$ and after that value oscillate with further increase in θ_0. The value of $|Z_5|$ for TP increases in the range $0^0 < \theta_0 < 70^0$ and then decreases with further increase in θ_0 .Due to viscosity the values for VSTP remain less than the values for TP. .The value for TP and VSTP are coincide when the angle of incidence is 90^0.

Fig. 4(f) shows that the behavior of variation of $|Z_6|$ is similar as that of $|Z_5|$ with difference in their magnitude values.

It is noticed from the fig.4(g) behavior of variation of $|Z_7|$ for TP and VSTP is similar with difference in magnitude. The values for TP are less than that of for VSTP in the whole range . These variations are shown in fig. 4(g).The values for TP and VSTP are magnified by multiplying by 10.

Fig. 4(h) shows that the behavior of variation of $|Z_8|$ for TP and for VSTP It is noticed that values of amplitude ratio for TP increase with the increase in the angle of incidence and oscillate for small range. The values of amplitude ratio for VSTP intially remains constant as θ_0 increases after that values of amplitude ratio increase slowly as the angle of incidence increases. The values of amplitude ratio for TP remain more than the values for VSTP in the whole range which reveals the effect of viscosity.

VIII. CONCLUSION

The study of reflection and transmission phenomena is a significant problem of continumm mechanics
In the present investigation, the plane waves propagation at the boundary surfaces of two micropolar viscothemoelastic solid half space with phase lags has been investigated The three phase lags theory of micropolar viscothermoelasticity has been used to study the problem The amplitude ratio of various reflected

and transmitted waves are obtained due to the incident of LD,thermal and CD waves. The expressions for reflection and transmission coefficients of various reflected and transmitted waves have been derived for particular cases. It is observed that when LD-wave or T-wave is incident, the behavior of variation of amplitude ratios for incident and tramsmitted CD-I and CD-II waves is similar. It is seen that when LD wave is incident, viscosity effect increases the values of amplitude ratios $|Z_1|$, $|Z_2|$,$|Z_6|$, $|Z_8|$for three-phase-lags . When T-waves is incident, viscosity effect decreases the value of amplitude ratio $|Z_3|,|Z_4|,|Z_7|,|Z_8|$ for three phase lag. When CD waves are incident,viscosity effect decreases the values of amplitude ratios $|Z_4|,|Z_5|,|Z_8|$.for three phase lags. The resultant quantities depicted graphically and observed are very sensitive towards viscosity,micropolarity and phase lag parameters .It is observed that amplitude ratio are the function of angle of incidence and frequency of waves.

Appriciable effect of viscosity has been observed on various reflected and transmitted waves. Figures shows that viscosity has oscillatory effect on numerical physical quantities obtained after computation process. The result obtained should be beneficial for the people working in micropolarviscoelastic with phase lags model.Although the problem is theoretical but it is significant in geophysical,geomechanics and geoengineering

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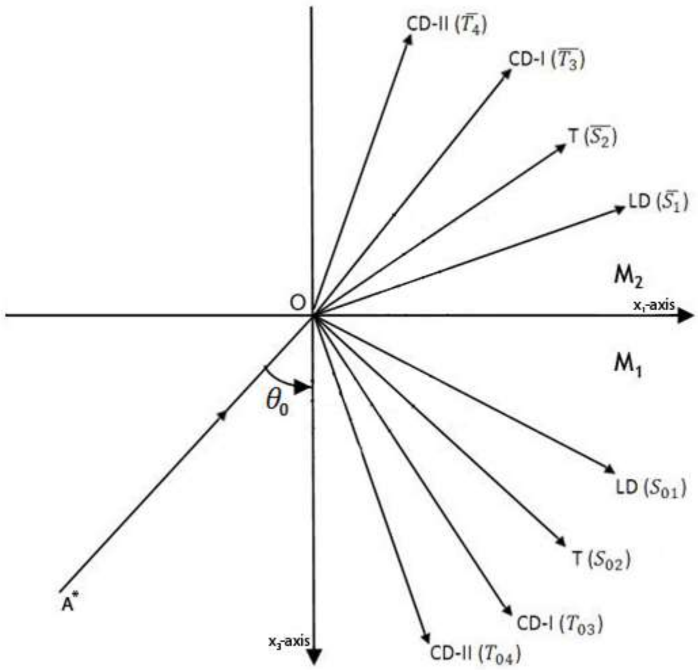

Fig. 1 Geometry of the problem

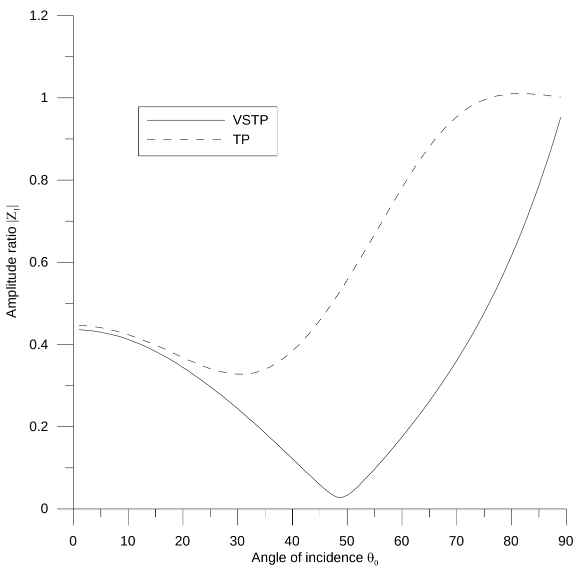

Fig. 2(a)

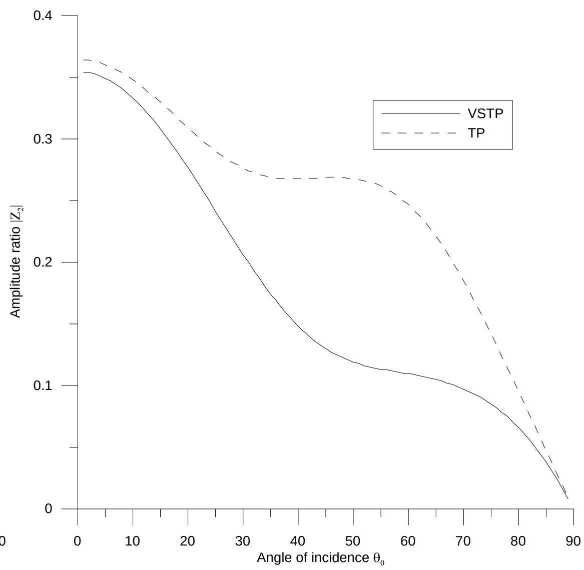

Fig.2(b)

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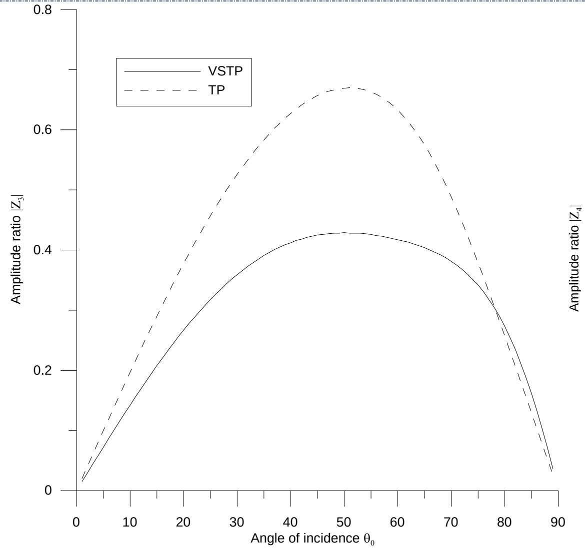


Fig. 2(c)

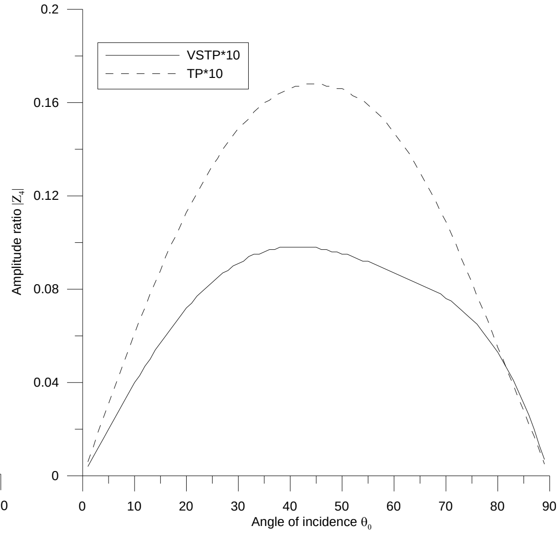


Fig.2(d)

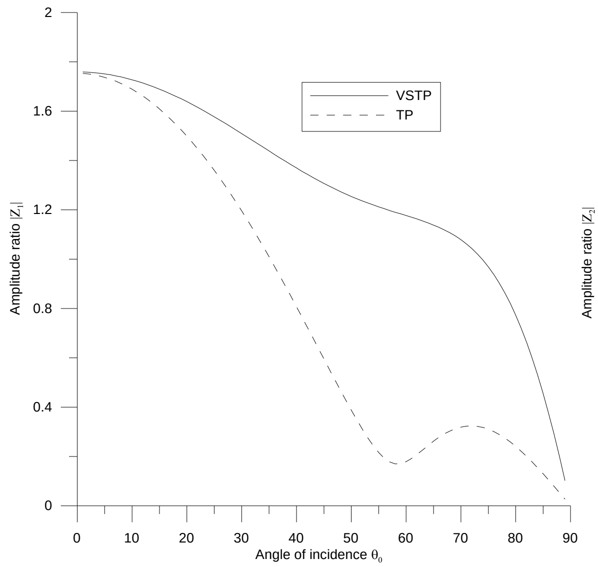


Fig.2(e)

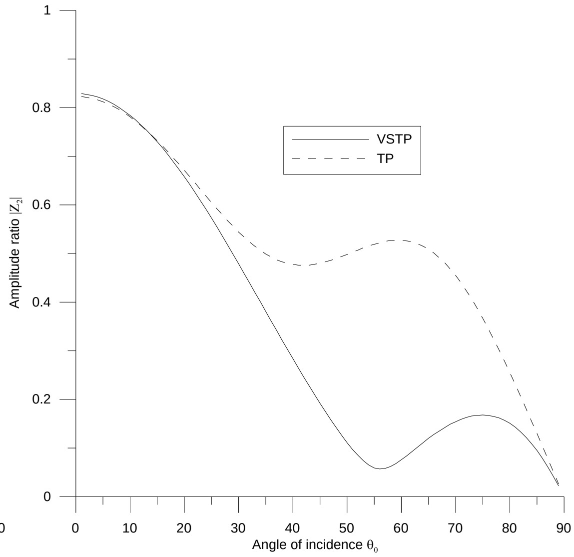
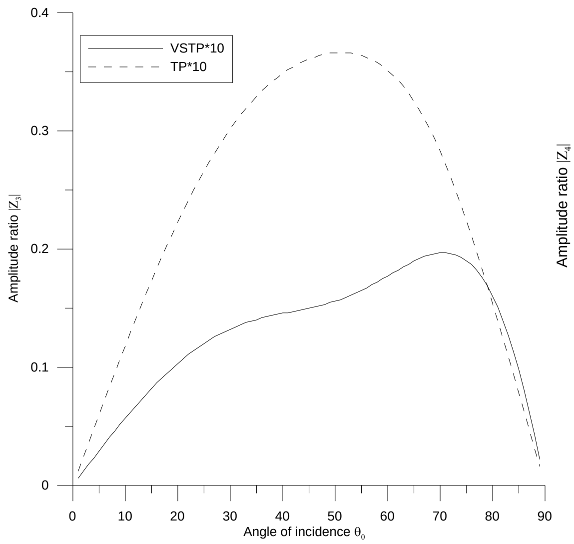


Fig.2(f)



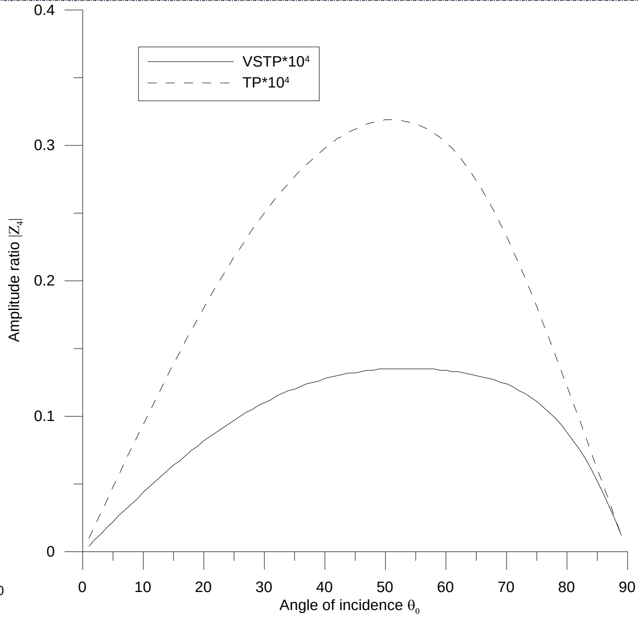


Fig. 2(g) ***Fig.2(h)***

Figs. 2(a)-2(h) Variations of amplitude ratios with angle of incidence for LD-wave

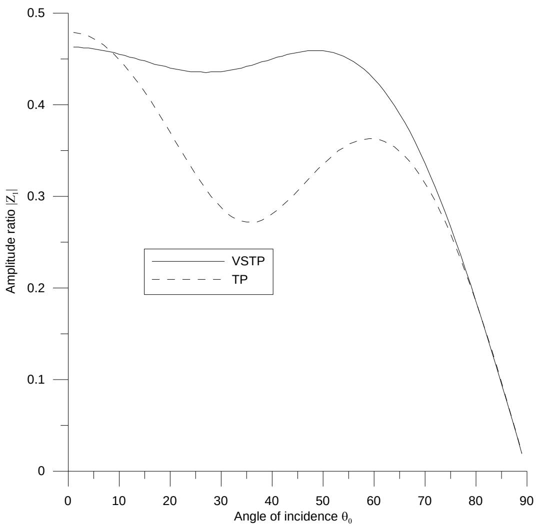


Fig. 3(a)

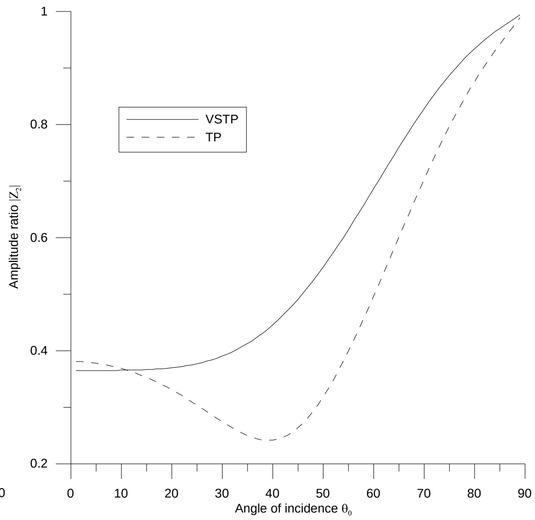


Fig.3(b)

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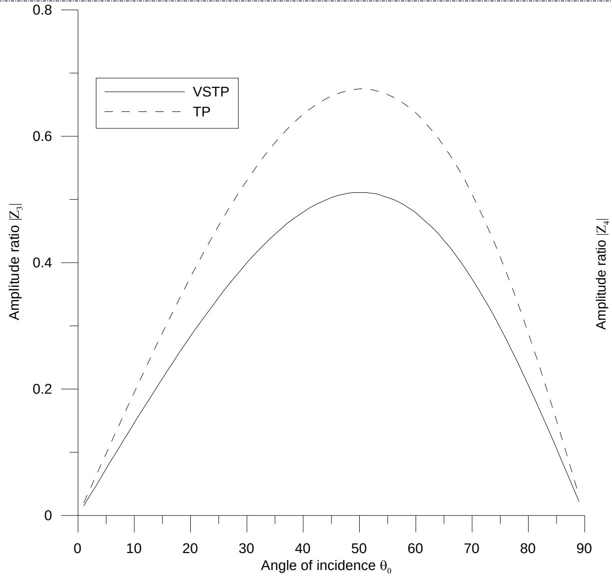


Fig. 3(c)

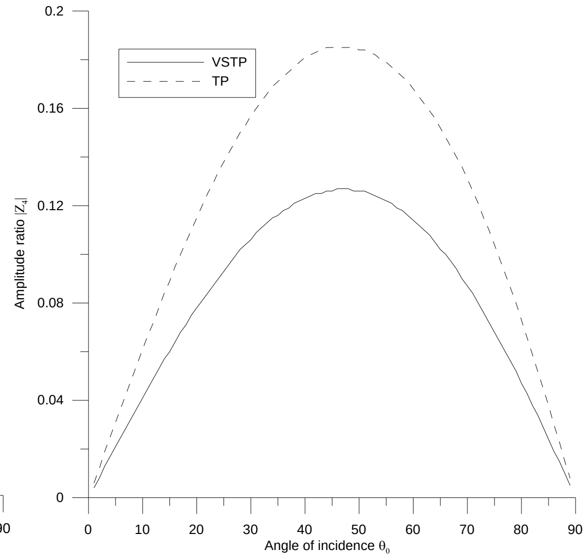
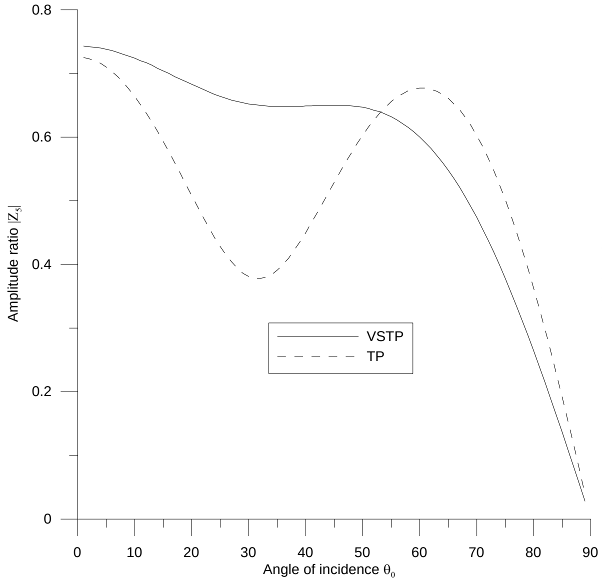


Fig. 3(d)



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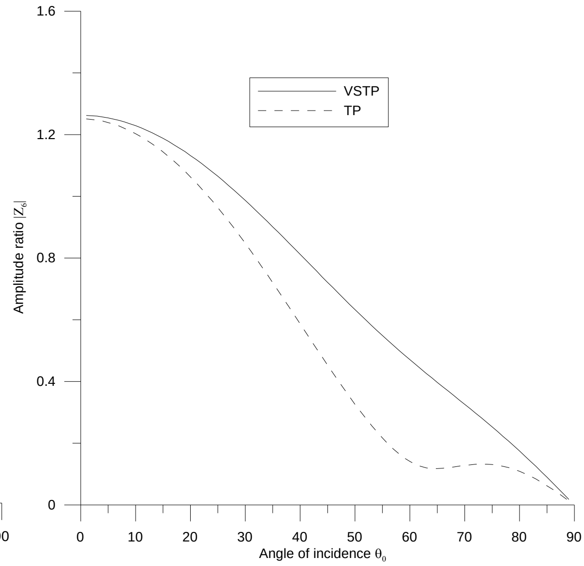

Fig.3(f)

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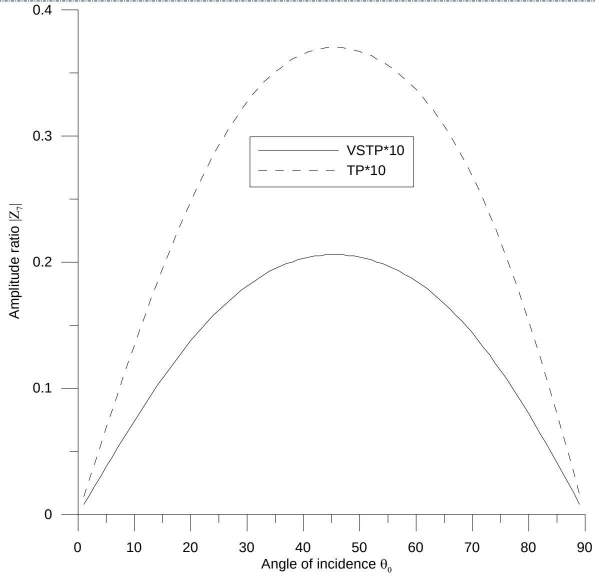

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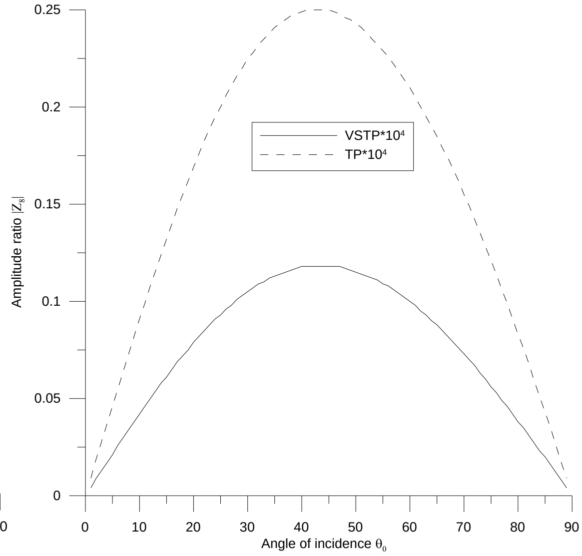

Fig. 3(h)

Figs. 3(a)-3(h) Variations of amplitude ratios with angle of incidence for T-wave

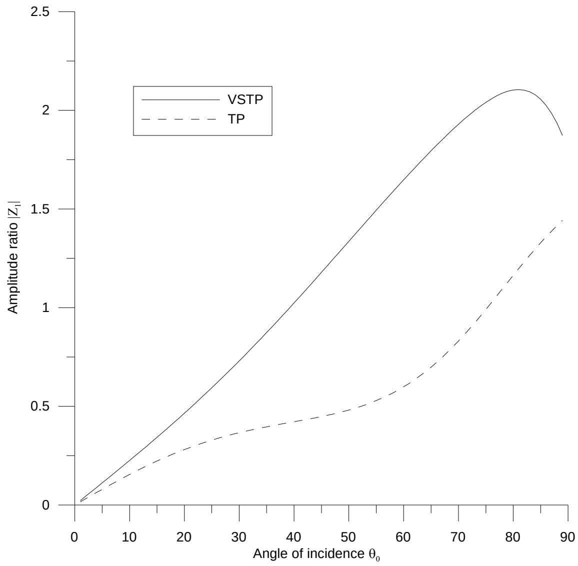

Fig. 4(a)

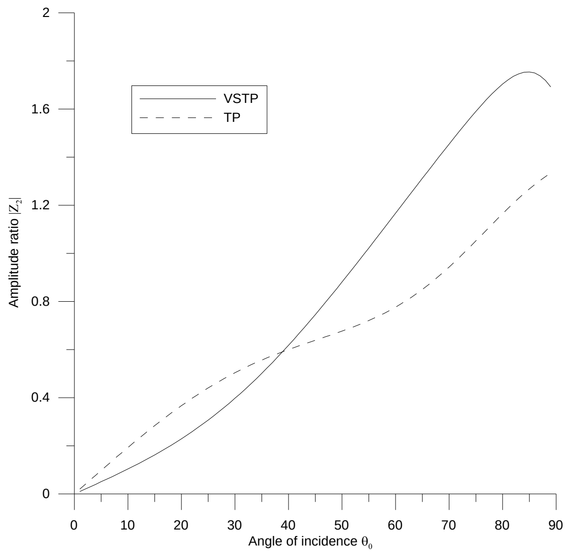

Fig. 4(b)

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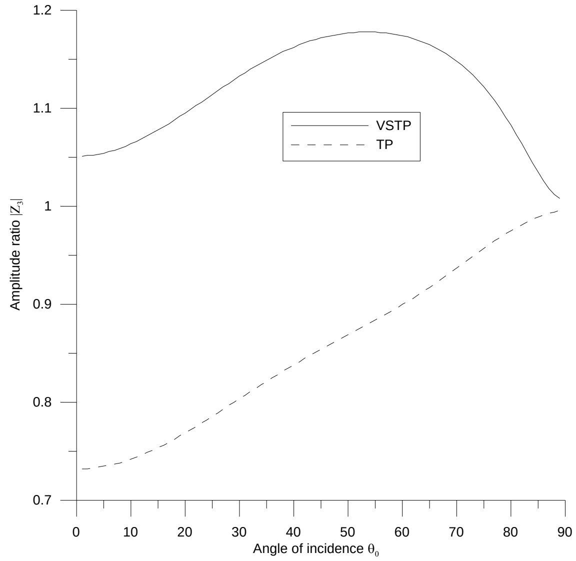


Fig. 4(c)

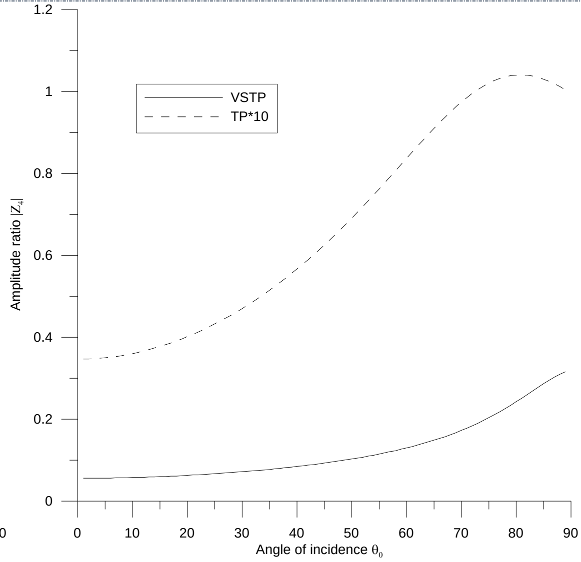
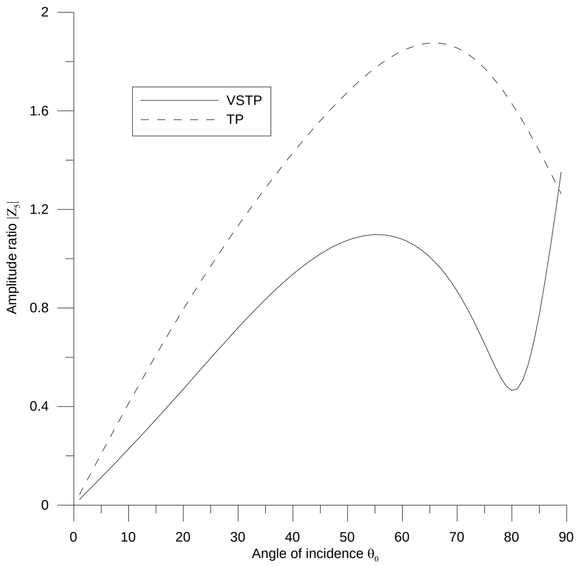


Fig. 4(d)



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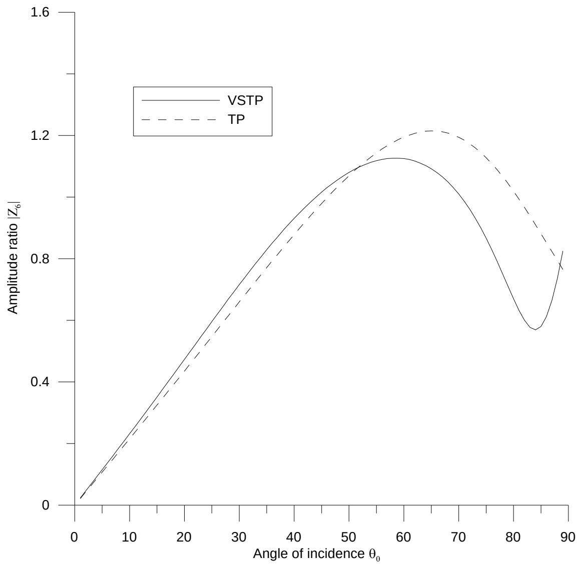
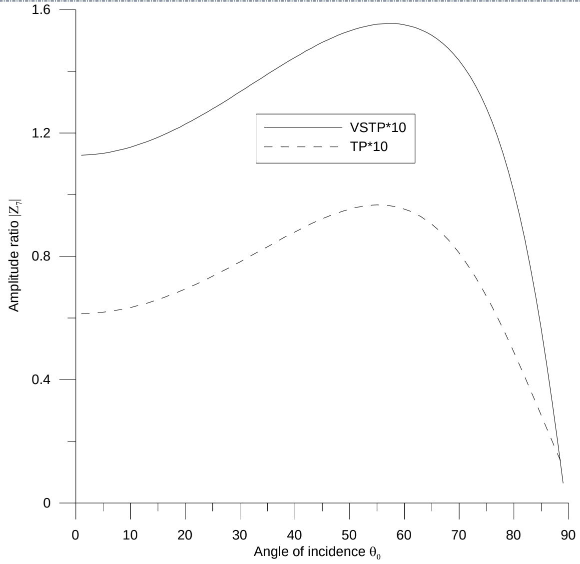


Fig. 4(f)



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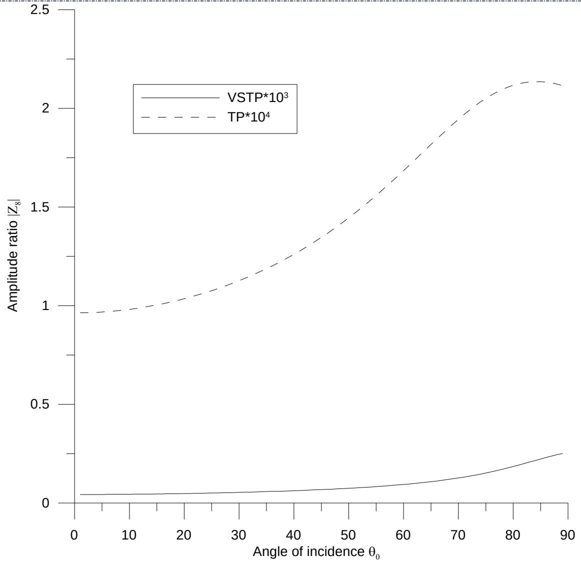

Fig. 4(h)

Figs. 4(a)-4(h) Variations of amplitude ratios with angle of incidence for CD-I wave

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